<u>In complete sentences, using proper English and mathematical notation,</u> state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: _____ / 5 PTS

IF f IS CONTINUOUS ON $[a,b]$
AND $A(x) = \int_a^x f(t)\,dt$
THEN $A'(x) = f(x)$ ON (a,b)

IF f IS CONTINUOUS ON $[a,b]$
AND $F'(x) = f(x)$ ON $[a,b]$
THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

GRADED BY ME

IF F' IS CONTINUOUS ON $[a,b]$
THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

Joe is moving from San Jose to San Francisco. If $f(r)$ is the fuel usage of Joe's moving truck (in liters per kilometer) when it is r kilometers from San Jose, what is the meaning of the equation $\int_4^7 f(r)\,dr = 2$?

SCORE: _____ / 2 PTS

NOTES: **Your answer must use all three numbers from the equation, along with correct units.**
Your answer should NOT use "x", "f", "integral", "antiderivative", "rate", "change" or "derivative".
Your answer should sound like normal spoken English.

JOE'S TRUCK USED 2 LITERS OF FUEL
TO DRIVE FROM THE POINT 4 KM FROM SAN JOSE
TO THE POINT 7 KM FROM SAN JOSE

GRADED BY ME

$$\int_{-1}^{1} \frac{3e^{2x}}{1+16e^{4x}}\,dx$$

$\boxed{u = 4e^{2x}}$ 1

$du = 8e^{2x}\,dx$

$\frac{3}{8}du = 3e^{2x}\,dx$

$x = 1 \to u = 4e^{2}$

$x = -1 \to u = 4e^{-2}$

1

$$= \boxed{\int_{4e^{-2}}^{4e^{2}}} \boxed{\frac{\frac{3}{8}}{1+u^2}\,du} \; 1$$

$$= \frac{3}{8}\boxed{\arctan u}\Big|_{4e^{-2}}^{4e^{2}}$$

1

$$= \boxed{\frac{3}{8}\left(\arctan 4e^{2} - \arctan 4e^{-2}\right)}$$

$\frac{1}{2}$

$$\int \frac{(5\sqrt{z} - 2z^3)^2}{3z^7}\, dz$$

$$= \int \frac{25z - 20z^{\frac{7}{2}} + 4z^6}{3z^7}\, dz$$

$$= \underbrace{\int \left(\frac{25}{3} z^{-6} - \frac{20}{3} z^{-\frac{7}{2}} + \frac{4}{3} z^{-1} \right) dz}_{1}$$

$$= \frac{25}{3} \cdot -\frac{1}{5} z^{-5} - \frac{20}{3} \cdot \frac{-2}{5} z^{-\frac{5}{2}} + \frac{4}{3} \ln|z| + C$$

$$= \underbrace{-\frac{5}{3} z^{-5}}_{1} + \underbrace{\frac{8}{3} z^{-\frac{5}{2}}}_{1} + \underbrace{\frac{4}{3} \ln|z|}_{1} + \underbrace{C}_{\frac{1}{2}}$$

MUST HAVE ABSOLUTE VALUE

$$\int_{-1}^{1} \frac{t^3+16t}{16-8t^2+t^4}\,dt$$

$16-8t^2+t^4=0$

$(4-t^2)^2=0$

$t=\pm 2$

1/2

INTEGRAND IS CONTINOUOUS ON $[-1,1]$

1

$$\frac{(-t)^3+16(-t)}{16-8(-t)^2+(-t)^4} = \frac{-t^3-16t}{16-8t^2+t^4}$$

$$= -\frac{t^3+16t}{16-8t^2+t^4}$$

INTEGRAND IS ODD

SO, INTEGRAL $= 0$ 1/2

$$\int_{-\pi}^{\pi} \frac{3\sin\theta}{1+2\cos\theta}\, d\theta$$

$1+2\cos\Theta=0$

$\cos\Theta=-\frac{1}{2}$

$\Theta=\frac{2\pi}{3}$

$\frac{1}{2}$ INTEGRAND IS NOT CONTINUOUS @ $\Theta=\frac{2\pi}{3}$ 1

FTC 2 DOES NOT APPLY

$\frac{1}{2}$

If $k(x) = \int_{x^5}^{\tan^{-1} x} \cos t^3 \, dt$, find $k'(x)$. **SCORE: _____ / 4 PTS**

$$k'(x) = \frac{d}{dx}\left[\int_{x^5}^{0} \cos t^3 \, dt + \int_{0}^{\tan^{-1}x} \cos t^3 \, dt\right]$$

$$= \frac{d}{dx}\left[-\int_{0}^{x^5} \cos t^3 \, dt + \int_{0}^{\tan^{-1}x} \cos t^3 \, dt\right]$$

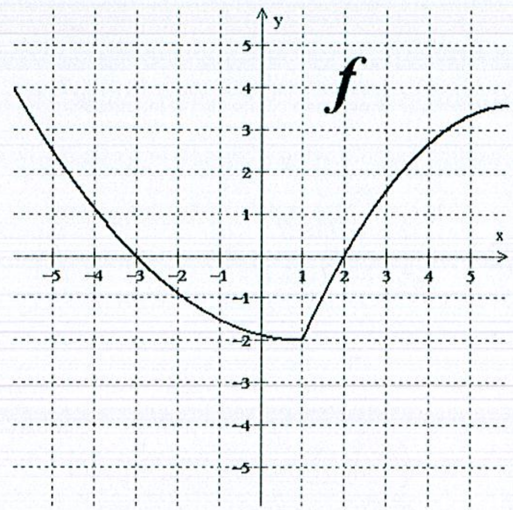
$$= -\frac{d}{dx^5}\int_{0}^{x^5} \cos t^3 \, dt \cdot \frac{dx^5}{dx} + \frac{d}{d(\tan^{-1}x)}\int_{0}^{\tan^{-1}x} \cos t^3 \, dt \cdot \frac{d(\tan^{-1}x)}{dx}$$

$$= -\cos(x^5)^3 \cdot 5x^4 + \cos(\tan^{-1}x)^3 \cdot \frac{1}{1+x^2}$$

$$= \underbrace{-}_{\frac{1}{2}}\underbrace{5x^4}_{1}\underbrace{\cos x^{15}}_{1} + \overbrace{\frac{\cos(\tan^{-1}x)^3}{\underbrace{1+x^2}_{1}}}^{\frac{1}{2}}$$

Let $R(x) = \int_{-5}^{x} f(t)\, dt$, where f is the function whose graph is shown on the right. SCORE: _____ / 6 PTS

[a] Write "I UNDERSTAND" to indicate that you understand that the graph shows f, but that the questions below ask about R.



[b] Find $R'(1)$. **Explain your answer very briefly.**

$R'(1) = f(1) = -2$

1 1

[c] Find all intervals over which R is concave down. **Explain your answer very briefly.**

$f = R'$ IS DECREASING ON $[-6, 1]$

1 1

[d] Find the $x-$coordinates of all local minima of R. **Explain your answer very briefly.**

$f = R'$ CHANGES FROM NEGATIVE TO POSITIVE AT $x = 2$

1 1